Johnson Hall, Room 106

## CALCULUS HANDOUT FOR DERIVATIVES

## I. Power rule

When doing the power rule, bring down the exponent of the variable and then multiply it with the current coefficient that is attached with the variable. The remaining exponent will be subtracted by 1 . Take a note that the derivative of a variable (let's say x ) is written as $\frac{d}{d x}$ or $\mathrm{x}^{\prime}$.

Formula: $a x^{n}=n * a x^{n-1}$ ( x is the variable, n and a are constants)

## II. Product Rule

The Product rule contains the power rule and has multiplication and addition involved. This applies when you take the derivative of two functions that are being multiplied together. You name one of the functions as your $f$ and the other is $g$. Take the derivative of $f$, times $g$, then add it with the derivative $g$, times $f$.

Formula: $(f * g)^{\prime}=f^{\prime} * g+f * g^{\prime}$
Note: You can freely pick which of the functions is your $f$ and $g$ (Yaaaaaaaay!!!) but avoid mixing them up once you're applying the product rule.

## III. Quotient Rule

The Quotient rule is used when taking the derivative of two functions that are being divided (usually in a form of a fraction).

Formula: $\left(\frac{f}{g}\right)^{\prime}=\frac{f^{\prime} * g-g^{\prime} * f}{g^{2}}$
Note: Unlike the product rule, you can't pick your own fand g. The dividend (numerator) is usually denoted as our f while the divisor (denominator) is denoted as our g .

## IV. Chain Rule

Chain rule involves getting the derivative of a composite function. Applying the chain rule starts from getting the derivative of the outmost function and then multiplying the whole function by the derivative of the inner function. Remember that chain rule applies on any other rule like product rule and quotient rule.

Formula: $(f(g))^{\prime}=f^{\prime}(g) * g^{\prime}$
Note: We usually leave the answer in factored form. Only solve for the equation when needed for a problem or when asked to.

## CALCULUS DERIVATIVE EXAMPLES

## Power Rule:

1.) $\frac{d}{d x} x^{3}=3 x^{2}$
2.) $\frac{d}{d x} 8 x^{5}=5 * 8 x^{4}=40 x^{4}$
3.) $\frac{d}{d x} 9 x^{\frac{1}{3}}=\frac{1}{3} * 9 x^{-\frac{2}{3}}=3 x^{-\frac{2}{3}}$
4.) $\frac{d}{d x} 3 x^{-2}=(-2) * 3 x^{-3}=-6 x^{-3}$

## Product Rule:

1.) $\frac{d}{d x}\left(x^{2}+3\right)\left(x^{4}-1\right)$

Let's say $\mathrm{f}(\mathrm{x})=x^{2}+3$ and $\mathrm{g}(\mathrm{x})=x^{4}-1$

$$
\mathrm{f}^{\prime}(\mathrm{x})=2 \mathrm{x} \quad \mathrm{~g}^{\prime}(\mathrm{x})=4 x^{3}
$$

Then

$$
\begin{aligned}
& =(2 x)\left(x^{4}-1\right)+\left(x^{2}+3\right)\left(4 x^{3}\right) \\
& =2 x^{5}-2 x+4 x^{5}+12 x^{3} \\
& =6 x^{5}+12 x^{3}-2 x
\end{aligned}
$$

## Quotient Rule:

1.) $\frac{d}{d x} \frac{x^{2}-3}{2 x^{4}}$
$=\frac{\left(4 * 2 x^{3}\right)-(2 x)\left(2 x^{4}\right)}{\left(2 x^{4}\right)^{2}}$
$=\frac{\left(8 x^{3}\right)\left(x^{2}-3\right)-(2 x)\left(2 x^{4}\right)}{4 x^{8}}$
$=\frac{8 x^{5}-24 x^{3}-4 x^{5}}{4 x^{8}}$
$=\frac{4 x^{5}-24 x^{3}}{4 x^{8}}$
2.) $\frac{d}{d} \frac{1}{d} \frac{x^{5}+3}{3 x^{2}-6}$
$=\frac{(2 * 3 x)\left(\frac{1}{5} x^{5}+3\right)-\left(5 * \frac{1}{5} x^{4}\right)\left(3 x^{2}-6\right)}{\left(3 x^{2}-6\right)^{2}}$
$=\frac{(6 x)\left(\frac{1}{5} x^{5}+3\right)-\left(x^{4}\right)\left(3 x^{2}-6\right)}{9 x^{4}-36 x^{2}+36}$
$=\frac{\frac{6}{5} x^{6}+18 x-\left(3 x^{6}-6 x^{4}\right)}{9 x^{4}-36 x^{2}+36}$
$=\frac{\frac{6}{5} x^{6}+18 x-3 x^{6}+6 x^{4}}{9 x^{4}-36 x^{2}+36}$
$=\frac{-\frac{9}{5} x^{6}+6 x^{4}+18 x}{9 x^{4}-36 x^{2}+36}$

## Chain Rule:

1.) $\frac{d}{d x}\left(x^{4}+1\right)^{2}$

Outer function is the (___) $)^{2}$ so

$$
=2 *\left(x^{4}+1\right)
$$

Then we multiply by the derivative
of the inner function which is 4 x

$$
\begin{aligned}
& =2^{*}\left(x^{4}+1\right)(4 x) \\
& =2\left(x^{4}+1\right)(4 \mathrm{x}) \\
& =8 \mathrm{x}\left(x^{4}+1\right) \\
& =8 x^{5}+8 x
\end{aligned}
$$

2.) $\frac{d}{d x}\left(2 x^{-3}+4\right)\left(5 x^{2}-3\right)$
$=(-3)\left(2 x^{-4}\right)\left(5 x^{2}-3\right)+\left(2 x^{-3}+4\right)(10 x)$
$=\left(-6 x^{-4}\right)\left(5 x^{2}-3\right)+20 x^{-2}+40 x$
$=-30 x^{-2}+18 x^{-4}+20 x^{-2}+40 x$
$=40 x-10 x^{-2}+18 x^{-4}$
2.) $\frac{d}{d x} 5\left(3 x+8 x^{3}\right)^{4}$
$=4 * 5\left(3 x+8 x^{3}\right)^{3} *\left(3+3 * 8 x^{2}\right)$
$=20\left(3 x+8 x^{3}\right)^{3}\left(3+24 x^{2}\right)$
You don't have to distribute further

Wasn't that fun???!!!!! :)

