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HANDOUT FOR SOLVING EXPONENTIALS AND LOGARITHMS

Logarithmic to Exponential:

If $y = \log_b(x)$ then

$b^y = x$

NOTE:

It is important to remember these statements are equivalent when working with logarithms

log(x)

Operator Operand

The operand for a logarithm is strictly positive: x>0

Example 1: if $\log_4(x) = 2$ then

$4^2 = x$

$16 = x$

Changing the statement from Exponential to Logarithmic is just working backwards

Example 2: if $4^2 = 16$ then

$\log_4 16 = 2$

When we are given a logarithm whose base isn't stated, the base is always 10

Example 3: $\log(x) = 2$ can be rewritten as: $\log_{10}(x) = 2$

Then we can solve for x by rearranging the statement to its equivalent exponential form:

So, $\log(x) = 2$ then becomes

$x = 10^2$

$x = 100$

When dealing with natural logs (ln), we approach and solve these problems in the same way. The only difference is that natural logs have a base "e" which is just a number approximately equal to 2.718. So when you see "ln(x)", there's a base of e.

Example 4: $\ln(x) = 2$ can be rewritten as $\log_e(x) = 2$, which then can be rearranged into exponential form as: $x = e^2$
NOTE: $\log_e(x) = ln(x)$

Here are some important logarithmic properties:

1. $\log_b(MN) = \log_b(M) + \log_b(N)$ 2. $\log_b(\frac{M}{N}) = \log_b(M) - \log_b(N)$ 3. $\log_b(M)^a = a log_b(M)$

4. $e^{\ln(x)} = x$ 5. $\ln(e)^x = x$ 6. $b^{\log_b(x)} = x$ 7. $\log_b(b)^x = x$ 8. $\log_b(x) = \frac{\log(x)}{\log(b)}$ 9. $log(1) = 0$

For properties 4,5,6, and 7, we're dealing with logarithms and exponentials composed with their inverses. When that happens, the logarithm and the exponential cancel and we're left with what they were operating on. Remember that $(f(f^{-1}(x))) = x \text{ and } (f^{-1}(f(x))) = x$

We can use this to our advantage when we come across problems like this:

$$log(x) + log(2) = log(5)$$

First we would start by combining the first terms with the logarithmic product rule (1.)

$$\log(2x) = \log(5)$$

Then, we can eliminate the logs by introducing the value of the base to both sides.

$$10^{\log(2x)} = 10^{\log(5)}$$

The log and the 10 cancel, leaving us with

$$2x = 5$$

Now we can solve for x

$$x = \frac{5}{2}$$

EXAMPLE PROBLEMS

Solving for X:

$ln(3x + 10) = 0$

(Property 4)

$e^{ln(3x+10)} = e^0$

$\cancel{e^{ln}}{}^{(3x+10)} = 1$

$3x + 10 = 1$

$3x = -9$

$x = -3$

Expanding:

$log\left(\frac{x}{yz}\right)^2$

(Property 3)

$= 2log(\frac{x}{yz})$

(Property 2)

$= 2\,log(x) - 2log(yz)$

(Property 1)

$= 2\,log(x) - 2[log(y) + log(z)]$

(Distributive Property)

$= 2\,log(x) - 2\,log(y) - 2\,log(z)$

Finding Exact Value:

$log\_2(120) - log\_2(3) - log\_2(5)$

(Property 2)

$= log_2(120) - log_2(3 * 5)$

(Property 2)

$= log_2\left(\frac{120}{15}\right)$

$= log_2(8)$

$= log_2(2)^3$

(Property 7)

$= 3$