

Domains of Functions

When looking for the domain of a function the first thing to look for are any restrictions the function may have. Domain restrictions arise when a function has “x” in the denominator, an even root, or logarithms. When writing the domain of a function we usually use interval notation.

- ♣ The first restriction is when a function has “x” in the denominator, the denominator cannot equal zero. To find the restrictions, we need to set the denominator not equal to zero and solve for “x.”

Example:

Find the domain of $f(x) = \frac{3x+1}{4x+2}$ using interval notation.

Since we have “x” in the denominator we set the denominator not equal to zero and solve for “x.”

$$4x + 2 \neq 0 \qquad 4x \neq -2 \qquad x \neq -\frac{2}{4} \qquad x \neq -\frac{1}{2}$$

So, the domain of $f(x)$ is all real numbers except when $x = -\frac{1}{2}$ in interval notation we say $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$

- ♣ The second restriction is when a function has “x” in the radicand of an even root $^{\text{Even}}\sqrt{\text{Radicand}}$. We look at the exponent to determine whether a root is even or not. If the root is even, then the radicand (what’s under the root symbol) must be greater than or equal to zero.

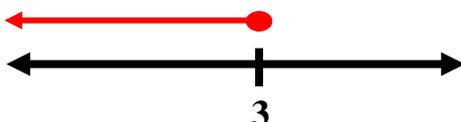
Example:

Find the domain of $f(x) = 3 - \sqrt{6 - 2x}$ using interval notation.

This function has a root so we look at the exponent of the root to determine if it is even. If the root (also called Radical) does not have an exponent similar to this example it is understood to be even. Since it is even we set the radicand greater than or equal to zero and solve for “x.”

$$6 - 2x \geq 0 \qquad 6 \geq 2x \qquad \frac{6}{2} \geq x \qquad 3 \geq x \qquad x \leq 3$$

Now, make a number line, and shade everything less than three with a closed dot at three.



In interval notation we say, $(-\infty, 3]$

- ♣ The third restriction is when we have logarithms operating on “x.” Whatever a logarithm is operating on is called the operand $\log(\text{operand})$. When we run into this restriction we set the operand strictly greater than zero and solve for “x.”

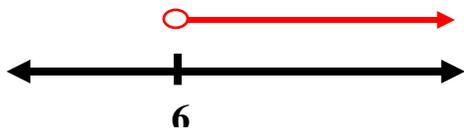
Example:

Find the domain of $f(x) = \log(2x^3 - 432)$ using interval notation.

Since, we have a logarithm with “x” in the operand we set the operand strictly greater than zero.

$$2x^3 - 432 > 0 \quad 2x^3 > 432 \quad x^3 > 216 \quad x > 6 \quad 6 < x$$

Again we should make another number line to visualize the domain.



Again, in interval notation we say $(6, \infty)$.

Here are some examples for you to work now.

Find the domain of each using interval notation.

$$f(x) = \frac{\sqrt{x+4}}{x-4}$$

$$f(x) = \frac{9}{x-6}$$

$$f(x) = 3\sqrt{x-2}$$

$$f(x) = 5\log(3x-27) + 16$$