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How to find possible rational zeros of polynomials

Here is an example: $f(x) = 2x^4 + x^3 + 17x^2 + 9x - 9 = 0$

First, we need to find the number of positive and negative real zeros of the polynomial.

To find the positive ones, we count the number of times the signs change in f(x) and subtract 2 from this number until we reach either 1 or 0.

We have + + + + -

The signs change only once, so there is 1 possible positive real zero in this polynomial.

Now to find the positive ones! We follow the same process except we find the number of sign changes of f(-x). So then we have $f(-x) = 2(-x)^4 + (-x)^3 + 17(-x)^2 + 9(-x) - 9 = 0$. So we have $f(-x) = 2x^4 - x^3 + 17x^2 - 9x - 9 = 0$. Any variable raised to the power of an even number will remain the same. Only those that are odd will change to the opposite sign.

So we have + - + - -

The signs change 3 times. Remember to subtract 2 from this number until we reach either 1 or 0. This means we could have either 3 or 1 negative real zeros in this polynomial.

To see our possibilities more easily, we can construct a table. Since the degree of the polynomial is 4 (the largest degree of x in the polynomial f(x)), the total number of zeros will also be 4 (ALWAYS).

+	-	Non-real	Total
1	3	0	4
1	1	2	4

Now we can determine the possible real zeros. To do this, divide each of the factors of the constant (typically the last term in the polynomial, it is the "number" without a variable attached to it) by each of the factors of the leading coefficient (typically the first term in the polynomial, it is the number with the variable with largest degree attached to it). Be sure to use both the positive and negative values.

The constant in f(x) is 9, and the leading coefficient is 2.

$$\pm\frac{factors\ of\ constant}{factors\ of\ leading\ coefficient} = \pm\frac{1,3,9}{1,2}$$

So here are all the possible rational zeros of $f(x) = 2x^4 + x^3 + 17x^2 + 9x - 9 = 0$

$$\pm(1, 3, 9, \frac{1}{2}, \frac{3}{2}, \frac{9}{2})$$

WOO! THAT WAS WAY TOO MUCH FUN! ☺